VII. "On the Measurement of Electric Resistance." By Professor W. THOMSON, F.R.S. Received June 6, 1861.

Part I. *New Electrodynamic Balance for resistances of short bars or wires.*

In measuring the resistances of short lengths of wire by Wheatstone's Balance *, I have often experienced considerable difficulty in consequence of the resistances presented by the contacts between the ends of the several connected branches or arcs. This difficulty may generally be overcome by soldering or amalgamating the contacts, when allowable; but even with soldered connexions there is some uncertainty relating to the dimensions of the solder itself, when the wires tested are very short. When soldering was not admissible, I have avoided being led into error, by repeating the experiment several times with slightly varied connexions; but I have in consequence sometimes altogether failed to obtain results by either Wheatstone's or any other method hitherto practised, as for instance in attempting to measure the electric resistances of a number of metallic bars each 6 millimetres long and 1 millim. square section, which were put into my hands by Mr. Calvert of Manchester, being those of which he and Mr. Johnstone determined the relative thermal conductivities in their investigation published in the Transactions of the Royal Society for March 1858. I have thus been compelled to plan a new method for measuring electric resistances in which no sensible error can be produced by uncertainty of the connexions, even though made with no extraordinary care.

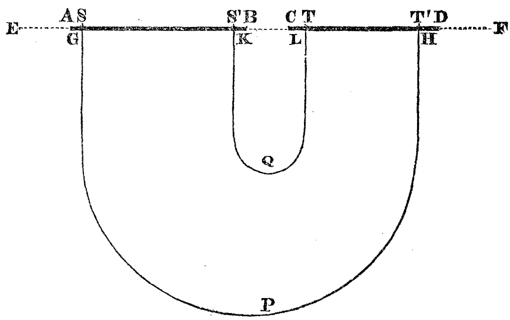
Let AB and CD be the standard and the tested conductors respectively. Let the actual standard of resistance be the resistance of the portion of AB between marks † S, S′ on it, and let it be required to find a portion TT′ of CD which has a resistance either equal, or bearing a stated ratio, to that standard.

Join BC either by direct metallic contact between them, or by

* I have given this name to the beautiful arrangement first invented by Professor Wheatstone, and called by himself a "differential resistance measurer." It is frequently called "Wheatstone's Bridge," especially by German writers. It is sometimes also, but most falsely, called "Wheatstone's Parallelogram."

† On the same principle as the "mètre à traits" instead of the "mètre à bouts" for a standard of length.

any ordinarily good metallic connexion with binding screws or otherwise; and join the two electrodes of a galvanic element to their other ends, A, D. Let GPH and KQL be two auxiliary conductors, which, to avoid circumlocutions, I shall call the primary and the



secondary testing-conductors respectively, with their ends applied to the marked points S, T′, S′, T. Let P and Q be points in these conductors to which the electrodes of the galvanometer are to be applied.

It is easily seen, and will be demonstrated below, that if the resistances of the testing-conductors be similarly divided in Q and P, and if their ends be in perfect conducting communication with the marked points of the main line to which they are applied, the condition that the galvanometer indication may be zero is that the ratio of the resistances of the standard and tested conductors must be the same as that in which the auxiliary conductors are each divided. Further, it is clear that by making the testing-conductors of incomparably greater resistances than any that can exist in the connexions at S, S′, T, T′, which can easily be done if these connexions are moderately good, the error arising from such imperfections as they must present may be made as small as is required*. To de-

* This method may be readily applied to Siemens's mercury standards (see Phil. Mag. Jan. 1861, or Poggendorff's 'Annalen,' 1860, No. 5), by introducing platinum wires through holes in the glass tube near its ends, as electrodes for the testing-conductors, and wires or plates of platinum at the ends, as electrodes for one pole of the battery and for connexion with the conductor to be compared with it, respectively. It will then not be the whole line of mercury from end to end, but the portion of it between the two platinum wires first mentioned, that will be the actual standard. The objection against the use of mercury as a standard of resistance, urged by Matthiessen, that the amalgamated copper

monstrate the above and to form an accurate idea of the operation of this method, it is necessary to investigate the difference of potentials (electromotive force) produced between Q and P, when a stated difference of potentials, E, is maintained between S and T′.

Let SS′, TT′ denote the resistances between the marks, on the standard and tested conductors respectively. Let GPH, GP, PH, KQL, KQ, QL denote the resistances of the testing-conductors and their parts according to the diagram, implying that

$$GPH = GP + PH, \text{ and } KQL = KQ + QL.$$

Let SG, HT′, S′K, LT be the resistances in the connexions at the marks; let

$$SG + GPH + HT' \text{ be denoted by } SPT',$$

and

$$S'K + KQL + LT \quad ,, \quad ,, \quad ,, \quad S'QT;$$

and let S′BCT denote the resistance between S′ and T composed of the resistance in the connexion and the resistances in the portions of

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electrodes which Siemens found necessary to give very perfect *end* connexions must render the mercury impure and increase its resistance sensibly after a time, is thus completely removed. It must be shown, however, that different specimens of commercial mercury, dealt with in the manner prescribed by Siemens, to remove impurities, shall always be found to have equal specific resistances, before his proposal to produce independent standards by filling gauged tubes with mercury can be admitted as valid. But the *transportation and comparison of actual standards* between different experimenters in different places is, and probably must always be, the only way to obtain *the most accurate possible common system of measurement*: and when a proper mutual understanding between electricians and national scientific academies, in all parts of the world, has been arrived at, as it is to be hoped it may be soon, through the assistance of the British Association and Royal Society if necessary, the use of definite metallic standards, whether the liquid mercury as proposed by Siemens, on the one hand, or the solid wire, alloy of gold and silver, on the other hand, proposed by Matthiessen (Phil. Mag. Feb. 1861), would be essential only in the event of all existing standards being destroyed.

Weber's absolute system is often referred to as if its object were merely to fix standards of resistance, and the difficulty and expense of applying it independently have been objected to as fatal to its general adoption. In reality its great value consists in the dynamic conditions which it fulfils, with relation to electro-magnetic induction, and to the mechanical theories of heat and of electro-chemical action. But it most probably will also be much more accurate than any definite metallic convention, for the re-establishment of a common metrical system, in case of the destruction of all existing standards.

tne main conductors from the marks S′ and T to their ends. Lastly, let R denote the resistance in the double channel $\left\{\begin{matrix}\mathrm{S'BCT}\\ \mathrm{S'QT}\end{matrix}\right\}$ between S′ and T. By the well-known principles of electric conduction, we have

$$\mathrm{R}=\frac{1}{\frac{1}{\mathrm{S'BCT}}+\frac{1}{\mathrm{S'QT}}}\quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

for the resistance in the double arc between S′ and T. Then, by addition, we have

$$\mathrm{SS'}+\mathrm{R}+\mathrm{TT'},$$

for the resistance from S to T′ by the channel SS′ $\left\{\begin{matrix}\mathrm{S'BCT}\\ \mathrm{S'QT}\end{matrix}\right\}$ TT′. This whole resistance is divided, by Q and its equipotential point in the direct channel S′BCT, into the parts

$$\mathrm{SS'}+\frac{\mathrm{S'Q}}{\mathrm{S'QT}}\cdot\mathrm{R},\ \text{and}\ \frac{\mathrm{QT}}{\mathrm{S'QT}}\cdot\mathrm{R}+\mathrm{TT'}.$$

Hence if, for simplicity, we suppose the potential at S to be 0, and at T′ to be E, and if we denote by q the potential at Q, we have

$$q=\mathrm{E}\,\frac{\mathrm{SS'}+\frac{\mathrm{S'Q}}{\mathrm{S'QT}}\cdot\mathrm{R}}{\mathrm{SS'}+\mathrm{R}+\mathrm{TT'}}\quad . \; . \; . \; . \; . \; . \quad (2)$$

Again, since P divides the resistance between S and T′, along the channel SPT′, into the parts SP and PT′, we have

$$p=\mathrm{E}\,\frac{\mathrm{SP}}{\mathrm{T'}}\quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

if we denote by p the potential at P. Hence

$$q-p=\mathrm{E}\,\frac{\mathrm{SS'}+\frac{\mathrm{S'Q}}{\mathrm{S'QT}}\cdot\mathrm{R}-\frac{\mathrm{SP}}{\mathrm{SPT'}}(\mathrm{SS'}+\mathrm{R}+\mathrm{TT'})}{\mathrm{SS}+\mathrm{R}+\mathrm{TT'}};$$

or, since $1-\frac{\mathrm{SP}}{\mathrm{SPT'}}=\frac{\mathrm{PT'}}{\mathrm{SPT'}}$,

$$q-p=\mathrm{E}\,\frac{\frac{\mathrm{PT'}}{\mathrm{SPT'}}\cdot\mathrm{SS'}-\frac{\mathrm{SP}}{\mathrm{SPT'}}\cdot\mathrm{TT'}+\mathrm{R}\left(\frac{\mathrm{S'Q}}{\mathrm{S'QT}}-\frac{\mathrm{SP}}{\mathrm{SPT'}}\right)}{\mathrm{SS}+\mathrm{R}+\mathrm{TT'}}\quad . \; . \quad (4)$$

Now let us suppose that, by varying one or more of the component

arcs in the balance-circuit, we reduce the galvanometer indication to zero, that is to say, make $q-p=0$. We shall have by equating the numerator of the preceding expression to zero, and resolving for TT′,

$$\mathrm{TT}'=\frac{\mathrm{PT}'}{\mathrm{SP}}\cdot\mathrm{SS}'+\mathrm{R}\left(\frac{\mathrm{SPT}'}{\mathrm{S'QT}}\,\frac{\mathrm{S'Q}}{\mathrm{SP}}-1\right). \quad . \quad . \quad . \quad (5)$$

To interpret this expression, it may be remarked that if the second term vanishes, that is to say, if

we have
$$\left.\begin{aligned}&\mathrm{R}\left(\frac{\mathrm{SPT}'}{\mathrm{S'QT}}\,\frac{\mathrm{S'Q}}{\mathrm{SP}}-1\right)=0\\&\mathrm{TT}'=\frac{\mathrm{PT}'}{\mathrm{SP}}\cdot\mathrm{SS}'\end{aligned}\right\}; \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and this is the condition aimed at in the arrangement. Now the connexions at S and T′ must be made so good that the resistance SG in the first is inappreciable in comparison with GP, and the resistance HT′, in the second, inappreciable in comparison with PH; so that we may have

$$\frac{\mathrm{PT}'}{\mathrm{SP}}\fallingdotseq\frac{\mathrm{PH}}{\mathrm{GP}},$$

where $\fallingdotseq$ denotes an equality not perfect, but having no appreciable error: and hence

$$\mathrm{TT}'\fallingdotseq\frac{\mathrm{PH}}{\mathrm{GP}}\cdot\mathrm{SS}'.$$

The condition

$$\mathrm{R}\left(\frac{\mathrm{SPT}'}{\mathrm{S'QT}}\cdot\frac{\mathrm{S'Q}}{\mathrm{SP}}-1\right)\fallingdotseq 0$$

is to be secured by one or other of two ways or by both combined; that is, by making

$$\mathrm{R}\fallingdotseq 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

or

$$\frac{\mathrm{SPT}'}{\mathrm{S'QT}}\,\frac{\mathrm{S'Q}}{\mathrm{SP}}\fallingdotseq 1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

or each as nearly as possible. If the connexion BC were quite perfect and the marks S′ and T were at the very ends of the conductors, the condition (a) would be fulfilled and there would be no necessity for the condition (b). We should then have a perfect Wheatstone balance,—the secondary testing-conductor $\left\{\begin{matrix}\mathrm{K}\\\mathrm{L}\end{matrix}\right\}$ Q becoming merely a part of the galvanometer electrode. Hence whenever the re-

sistance S′T can be made absolutely insensible, Wheatstone's balance leaves nothing to desire, provided the ends of the testing-conductor are applied to marked points on the standard and tested conductors, and the battery electrodes to their outer ends, or to points of them between their outer ends and those marked points. When, however, as very frequently is the case, S′T may be made small but not absolutely insensible in comparison with the resistances of the standard and tested conductors, the addition of the "secondary testing-conductor" becomes valuable, even if it be only arranged to give a rough approximation to the condition $\frac{SPT'}{S'QT} \frac{S'Q}{SP} = 1$ *, since it will reduce the error to the fraction $\frac{SPT'}{S'QT} \cdot \frac{S'Q}{SP} - 1$, of the small resistance R †. But further, when, as in experiments on short thick bars like those of Mr. Calvert, S′T cannot by any management be got to be small in comparison with TT′, the use of the secondary testing-conductor becomes essential, and the most accurate possible fulfilment of the condition

$$\frac{SPT'}{S'QT} \cdot \frac{S'Q}{SP} = 1$$

must be aimed at. This is to be done by dividing the secondary testing-conductor at Q, in very exactly the same ratio as the primary at P, and taking care that the resistances in the connexions S′K, LT are very small in comparison with KQ and QL.

Part II. *Suggestions for carrying out these principles in practice.*

When high accuracy is not required, the two "testing-conductors" may be made of wires stretched straight in parallel lines, and the connexions for the galvanometer electrodes may be applied to them by means of a slide on a graduated scale—as in one of the common forms of Wheatstone's balance, with sliding contact on single testing-conductor. This form is very objectionable, however, whether for Wheatstone's balance or the method I now propose: (1) because

* This of course is equivalent to SPT′ : SP : : S′QT : S′Q, and means that the secondary conductor is to be divided by one galvanometer electrode in the same proportion as the primary is divided by the other.

† In such cases R will, according to equation (1) above, be nearly equal to S′BCT, but somewhat less.

it is impossible to secure that the different parts of each testing-conductor shall be accurately at the same temperature; (2) because the resistances at the ends of the fine stretched wire or wires are always sensible in comparison with the smallest measured differences produced by the slide; (3) because the stretched wire itself is never of absolutely equal gauge throughout, and, even if sensibly so when first put into the instrument, soon ceases to be so in consequence of the friction of the sliding contact which it experiences in use*; (4) because, in even the hastiest experiments, provided a rationally planned galvanometer is used, a far higher proportional degree of accuracy is easily attained in measuring electrical resistances against a standard of resistance than can be at all attained, without very extraordinary precautions and the assistance of a microscope, in measuring lengths under a yard or two against a standard of length.

When the highest accuracy is required, I always use for primary testing-conductor the bisected conductor which I described to the British Association at its Glasgow meeting in 1855. This consists of a fine, very perfectly insulated wire, doubled on itself and wound on a bobbin, with very stout terminals soldered to its ends, and an electrode soldered to its middle, for joining to the galvanometer electrode. The two terminal and the middle electrodes thus attached to the testing-conductor, I have generally hitherto made flexible, either of thick wire, or strand of wires like the conductor of a submarine cable; but, for many applications, it is more convenient to make them solid metal blocks, with binding screws, insulated rigidly upon the bobbin which bears the conductor. The two halves into which the conductor is doubled must be very accurately equalized as to electric resistance when they are wound on the bobbin, and before the terminals are finally attached. This I find can be done with great accuracy; and when, after the terminals are soldered on, the electric bisection is once found perfect, it seems to remain so, without sensible change, for years. The close juxtaposition of the two branches of the testing-conductor on this plan ensures an almost absolute equality of temperature between them in all circumstances,

* This defect I have remedied by frequently putting in a new wire for testing-conductor in working with a sliding-scale Wheatstone's balance.

and thus renders easy a degree of accuracy in the measurement of resistances quite unattainable with any other form of Wheatstone's balance. In the new method which I now propose for low resistances, I make the secondary conductor on exactly the same plan, and generally of about the same dimensions, as the primary. The bisected testing-conductors are only available when the resistances of the standard and of the tested conductor can be made equal; and with them the method which has been described above seems to be the most accurate possible for testing a perfect equality of resistance between two conductors.

The same plan of testing-conductors seems still the best, even when testing by equality cannot be practised,—with only this difference, that the two branches of each testing-conductor, instead of being made of equal resistance, must be adjusted to bear to one another very exactly the ratio which the tested resistance is to bear to the standard. By proper care, to prevent the bobbin of either testing-conductor from getting any non-uniform distribution of temperature, great accuracy may still be secured; but it is scarcely possible to maintain so very close an agreement of temperature, and therefore so constant a ratio of resistances, as when the two branches are equal lengths of one wire coiled side by side.

The use of this plan of conductors divided in a fixed ratio, whether for the single testing-conductor in Wheatstone's balance, or for the primary and secondary testing-conductors in the new method now proposed, requires that either the standard or the tested conductor can be varied so as to adjust the resistance of one to bear precisely that ratio to the resistance of the other. In certain cases this may be done advantageously by shifting one or other of the contacts S, S', T, T' along the standard or the tested conductor, as the case may be. If, for instance, T or T' can be shifted conveniently, the object of the measurement may be to find by trial on the tested conductor a portion TT' from mark to mark, of which the resistance bears a stated ratio to the fixed standard SS' from mark to mark. But by far the easiest working, and in most cases the most accurate also, is to be done by means of a well-arranged series of standards with terminals adapted for combining them in such a manner as to give to a minute degree of accuracy whatever resistance may be required. In a future communication on standards of electric resistance, I

intend to describe plans for attaining this object through a wide range of magnitude (resistances from 10^5 to 10^{13} British absolute units of feet per second on Weber's invaluable system). In the mean time I shall merely say that I have formed a plan which I expect will prove very advantageous for low resistances, and which consists in combining the standards, whichever of them are required, in multiple arc (or "parallel" arcs, according to an expression sometimes used), so as to add their *conducting powers**,—instead of in series, as in all arrangements of resistance coils hitherto used, by which the *resistances* of the component standards are added.

Part III. *General Remarks on Testing by Electro-dynamic Balance.*

I shall conclude by remarking that the sensibility of the method which has been explained, as well as of Wheatstone's balance, is limited solely by the heating effect of the current used for testing. To estimate the amount of this heating effect, let e and f be the parts of the whole electromotive force, E, which act in the standard SS′, and tested conductor TT′ respectively; so that, in accordance with the notation used above, we have

$$\left.\begin{aligned} e&=\mathrm{E}\frac{\mathrm{SS}'}{\mathrm{SS}'+\mathrm{R}+\mathrm{TT}'} \\ f&=\mathrm{E}\frac{\mathrm{TT}'}{\mathrm{SS}'+\mathrm{R}+\mathrm{TT}'} \end{aligned}\right\} \quad \ldots\ldots\ (7)$$

of its substance. Following Weber, I define the resistance of a bar or wire one foot long, and weighing one grain, its *specific resistance*. It is much to be desired that the *weight-measure*, rather than the *diameter* or *the volume-measure*, should be generally adopted for accurately specifying the gauge of wires used as electric conductors.

With reference to either SS′ or TT′ (the first, for instance), let us use the following notation:—

l its length in feet;
w its mass per foot in grains;
s the specific heat of its substance;
σ the specific resistance of its substance.

* The reciprocal of the resistance of a "conductor" or "arc" I call its *conducting power*. The conducting power of a bar or wire of any substance one foot long and weighing one grain, I call the *specific conductivity of its substance*.

Thus, since we have taken SS′ to denote its actual resistance, we have

$$SS' = \frac{\sigma l}{w}.$$

Now, Weber's system of absolute measurement for electromotive forces and for resistances being followed, I have shown* that *the mechanical value of the heat generated per unit of time in any fixed conductor of uniform metallic substance is equal to the square of the electromotive force between its extremities, divided by its resistance.* This in the present case is equal to

$$\frac{e^2 w}{l\sigma};$$

and if J denote Joule's mechanical equivalent of the thermal unit, we therefore have

$$\frac{e^2 w}{Jl\sigma}$$

for the rate per second at which heat is generated in SS′. This will at first go entirely to raise its temperature†. Now wl is its mass in grains, and therefore wls is its whole thermal capacity; and if we divide the preceding expression by this, we find

$$\frac{e^2}{Jl^2 s\sigma}$$

for the rate per second at which it commences to rise in temperature at the instant when the battery is applied. If we call $\frac{e}{l}$ the electromotive force per foot, we may enunciate the result thus:

The rate at which a linear conductor of uniform metallic substance commences rising in temperature at the instant when an electric current commences passing through it, is equal to the square of the electromotive force per unit of length divided by the continued product of Joule's equivalent into the specific heat of the substance, into the specific resistance of the substance.

Let us suppose, for example, that the conductor in question is

* In a paper "On the Mechanical Theory of Electrolysis," Philosophical Magazine, Dec. 1851.

† As soon as it has risen sensibly in temperature it will begin to give out heat by conduction, or by conduction and radiation, to the surrounding matter; and the rate at which it will go on rising in temperature will be the rate expressed by the formula in the text (with the true specific resistance, &c., for each temperature), diminished by the rate of loss to the surrounding matter.

copper of best electric conductivity. Its specific resistance will be about 7×10^6, and its specific heat about $\cdot 1$. The value we must use for Joule's equivalent will be 32·2 times the number 1390, which Joule found for the mechanical value in foot-grains of the thermal unit Centigrade, since the absolute unit of force, being that force which acting on a grain of matter during a second of time generates 1 foot per second of velocity, is $\frac{1}{32\cdot 2}$ of the weight of a grain in middle latitudes of Great Britain. Thus we find

$$J = 44758.$$

Hence the expression for the rate in degrees Cent. per second, at which the temperature begins rising in a copper conductor, is

$$\frac{\left(\frac{e}{l}\right)^2}{313 \times 10^8}.$$

I have found the electromotive force of a single cell of Daniell's to be about $2\cdot 3 \times 10^6$ British absolute units* ; and if we suppose $\frac{1}{n}$ of this to go to each foot of the conductor in question, we shall have

$$\left(\frac{e}{l}\right) = \frac{2\cdot 3^2 \times 10^{12}}{n^2} = \frac{5\cdot 29 \times 10^{12}}{n^2};$$

and therefore the expression for the rate of heating becomes

$$1\cdot 69 \times \frac{100}{n^2}.$$

Now, by using a sufficiently large single cell, we may make the electromotive force, E, between S and T′, be as little short as we please of the whole electromotive force of the cell. We might then, in testing by equality, with a standard and a tested conductor each three inches or so long, and using a single cell, have nearly as much as half the electromotive force of one cell acting per quarter foot of these conductors, or two cells per foot. Hence if either is of best conductive copper, its temperature would commence rising at the rate of $4 \times 169°$ or $676°$ Cent. per second. It would be almost impossible to work with so high a heating effect as this. But if we use only $\frac{1}{10}$th of the supposed electromotive force, that is to say $\frac{1}{5}$th of a cell per foot of the copper conductor, the rate of heating will be reduced to $\frac{1}{100}$, that is to say, will be $6°\cdot 76$ per second. By using

* Proceedings of the Royal Society, February 1860.

only very brief battery applications, it would be possible to work with so high a rate of heating as that, without having the results much vitiated by it. But $\frac{1}{50}$ of a cell per foot will give only $\cdot0676^\circ$ of heating effect per second, and will be quite a sufficient battery power to use in most cases. In the case we have supposed, for instance, of conductors only three inches long, the electromotive force on each would then be about $\frac{1}{200}$ of the electromotive force of the cell. What we denoted above by e and f in equations (7) would therefore each have this value. Hence, by equation (4), we see that the effect of a difference of $\frac{1}{1000}$ between SS′ and TT′ would be to give $q-p$ the value $\frac{1}{400000}$ of the electromotive force of a single cell. Now one of the light mirror* galvanometers, which I commonly use, reflecting the image of a gas or paraffine lamp to a scale 25 inches distant, would, if made with a coil of 50 yards of copper wire of moderate quality, weighing 5 grains per foot, give a deflection of half a division of $\frac{1}{40}$ of an inch on this scale, with an electromotive force of $\frac{1}{4000000}$ of a single cell†. Hence by using such a galvanometer, and primary and secondary conductors of sufficient resistances to fulfil the condition of doing away with sensible error from imperfect connexions in the manner explained above, but yet of resistances either less than or not many times greater than the resistance of the galvanometer coil, it is easy to test to $\frac{1}{1000}$ the resistance of a copper wire or bar not more than 3 inches long. The

* The mirror is a circle of thin "microscope glass" about three-eighths of an inch in diameter, silvered in the ordinary manner; and a small piece of flat file steel of equal length, attached to its back by lac varnish, constitues the "needle" of the galvanometer. The whole weight of mirror and needle amounts to from 1 to $1\frac{1}{2}$ grain. It is suspended inside the galvanometer coil by single silk fibre about $\frac{1}{8}$ inch long. It is necessary to try many mirrors thus prepared, each with its magnet attached, before one is found giving a good enough image. I am much indebted to Mr. White, optician, Glasgow, for the skill and patience which he has applied to the very troublesome processes involved.

† In this state of sensibility the needle is under Glasgow horizontal magnetic force of the earth alone; and, with its mirror, it makes a vibration one way in about ·7 of a second. In many uses of my form of mirror galvanometer, both for telegraphic and for experimental purposes, I find it convenient to make its indications still more rapid, though, of course, less sensitive, by increasing the directive force by means of fixed steel magnets. On the other hand, I use fixed steel magnets to diminish the earth's directing force and make the needle more sensitive, when very high sensibility is wanted; but this would be inconvenient for the application described in the text, because effects of thermo-electric action would be made too prominent.

current we have found to be sufficient for this object would only produce a heating effect of $\cdot 14°$ in two seconds, which, with good apparatus, is more than enough of time, as I shall show presently. The influence of this heating effect may be regarded as nearly insensible, since even as much as $\cdot 2°$ only alters the resistance of copper by about $\frac{3}{4000}$.

In all measurements of electric resistance, whatever degree of galvanic power is used, a spring "make and break" key* ought to be placed in one of the battery electrodes, so that the current may never flow except as long as the operator wills to keep it flowing, and presses the key. I introduce a second similar spring key in one of the galvanometer electrodes (that is between either Q or P and the galvanometer coil), so arranged that the pressure of the operator's finger on a little block of vulcanite attached to either spring shall first make the contact of the first spring (completing the battery circuit), and when pushed a little further, shall make the contact of the second spring and complete the galvanometer circuit. The test for the balance of resistances will then be that not the slightest motion of the needle is observable as a consequence of this action on the part of the operator. The sensibility of the arrangement is doubled by a convenient reverser in the galvanometer circuit, by which the current, if any, may be reversed easily by the operator while keeping the two connexions made by full pressure on the double spring key just described. Another convenient reverser should be introduced into the battery circuit, to eliminate effects of thermo-electric action if sensible.

It may often happen, unless the galvanometer is at an inconveniently great distance from the conductors tested, that its needle will be directly affected to a sensible extent by the main testing-current; but with the arrangement I have proposed the observer tests whether or not this is the case by pressing the double spring-key to only its middle position (battery contact alone made), and watching whether or not the needle moves perceptibly. If it does not move perceptibly, he has nothing more to do than immediately to press the double key home, to test the balance of resistances. If the needle does move when the key is pressed to its middle position, he may, when in

* Morse's original telegraph key, which instrument-makers have "improved" into the in every respect worse form in which it is now commonly made—a massive contact-lever urged by a spring.

other respects allowable, keep the current flowing by holding the key in its middle position till the needle comes to rest, or at least till it shows the point towards which its oscillations converge, and then press home to test the balance of resistances. When the very highest accuracy is aimed at, or when, for any reason (as, for instance, extreme shortness in the standard or tested conductor), only the shortest possible duration of current is allowable, the position of the galvanometer, with reference to the battery and the other portions of circuit, must be so arranged that its needle may show no sensible deflection when the key is pressed to the middle position. Ignorant or inadvertent operators are probably often led into considerable mistakes in their measurements of resistance by confounding deflections due to direct electro-magnetic influence of battery, battery electrodes, or standard, tested, or testing-conductors, on the needle of the galvanometer, with the proper influence of a current through its own coil,—a confusion which can only be resolved by making or breaking the galvanometer circuit while the battery circuit is kept made, for which there is no provision in the ordinary plans of Wheatstone's balance. We may, however, suppose that most experimenters will be sufficiently upon their guard against error from such a source. But there is another and a much more important advantage in the double-break arrangement which I now propose. Electro-magnetic inductions will generally be sensible* in some or in all of the different branches of the compound circuit, and cannot, except in very special cases, be exactly balanced as regards electromotive force between P and Q with the arrangement which makes an exact balance of resistances. Hence, at the moment when the battery contact is made, there must generally be an electromotive impulse between Q and P, which will drive a current through the galvanometer coil, and make an embarrassing deflection of the needle if the galvanometer circuit is complete at that instant (as it is in the common plans of Wheatstone's balance), and will require the observer to wait until the needle comes to rest, or until he can tell

* I make them as little sensible as possible in my coiled testing-conductors, and in sets of coiled standards of resistance, by either doubling each coil or each branch of each coil on itself, or by reversing the lathe at regular intervals in winding on any single coil on a bobbin,—a plan which has also the advantage of rendering the direct electro-magnetic action of any coil so wound very small or quite insensible on any galvanometer needle in its neighbourhood.

precisely to what point its oscillations converge, the current being kept flowing all the time, before he can discover whether the balance of resistances has been attained or not. This absolutely precludes very refined testing, since, whether by the heating and consequent augmentation of resistance of some part of the balanced branches, or by thermo-electric reactions consequent on heating and cooling effects at junctions of dissimilar metals when the branches of the balance are not all of one homogeneous metal, or last, though not least, by the eye losing the precise position where the galvanomer needle or indicating image rested, it is not possible to use the full sensibility of the galvanometer for testing a zero if its needle is allowed to receive such a shock in the course of the weighing. Embarrassment from this source is completely done away with by using the double spring key described above, and giving time, from its first to its second contact, to allow the electro-magnetic induction to subside. An extremely small fraction of a second is enough in almost all cases; and the operator may therefore generally press the key home almost as sharply as he will or can. But when there is a large "electro-dynamic capacity"* in any part of the balance-circuit, as, for example, when the coil of a powerful electro-magnet with soft iron† core is the conductor whose resistance is tested, it may be necessary to keep the key in its middle position for a few seconds before pressing it home, to avoid obtaining what might be falsely taken for an

* This term I first introduced in a communication "On Transient Electric Currents" (Phil. Mag., June 1853), to designate what for any electric current through a given conductor is *identical* in meaning with the "simple-mass equivalent" in the motion of Attwood's machine as ordinarily treated. A rule for calculating the electrodynamic capacity is given in that communication; also the rule, with an example, in Nichol's Cyclopædia, article "Magnetism—Dynamical Relations of."

† Giving a resistance to the commencing, to the ceasing, or to any other variation in the strength of an electric current (precisely analogous to the effect of inertia on a current of common fluid),—which it seems quite certain must be owing to true inertia (not of what we should at present regard as the electric fluid or matter itself flowing through the conductor, but) of motions accompanying the current, chiefly rotatory with axes coinciding with the lines of magnetic force in the iron, air, and other matter in the neighbourhood of the conductor, and continuing unchanged as long as the current is kept unchanged. See Nichol's Cyclopædia, article "Magnetism—Dynamical Relations of," edition 1860; also Proceedings of the Royal Society, June 1856; or Phil. Mag., vol. Jan.–June 1857.

indication of too great a resistance to conduction (or "frictional" resistance, as I have elsewhere called it*), being a true indication of resistance or reaction of inertia to the commencement of the current in the electro-magnetically-loaded branch†. In such cases it is impossible, either by electrodynamic balance or in any other way, to obtain a measurement of resistance without keeping the battery applied for the few seconds required to produce sensibly its final strength of current undiminished by inductive reaction, over and above the time required to get an indication from the galvanometer. But, as already remarked, in all ordinary cases, the inductive reaction becomes insensible after a very small fraction of a second, and the operator may press the double key home to its second contact almost as sharply as he pleases. With such a galvanometer as I have described, he need not hold it down for more than ·7 of a second (the time of the simple vibration of the needle‡) to test the balance of resistances. The order of procedure will therefore generally be this:—The operator will first strike the key sharply, allowing it to rise again instantly, adjust resistances in the balance-circuit according to the indication of the galvanometer; strike the key sharply again, readjust resistances; and so on, until the balance is nearly attained. He will go on repeating the process, but holding the key down rather longer each time. At the last he will press the key gently down, hold it pressed firmly for something less than a second of time, and let it rise again; and if the spot of light reflected from the mirror of the galvanometer does not move sensibly, the resistances are as accurately balanced as he can get them.

* "Dynamical Theory of Heat, Part VI., Thermo-electric Currents," Transactions of the Royal Society of Edinburgh, 1854; and Phil. Mag. 1856.

† It is probable that a Wheatstone's balance, perfectly adjusted for equilibrium of resistances to conduction, and used with the galvanometer circuit constantly made, so as to show the whole effect of the inductive impulse, may afford the best means for making accurate metrical investigations on electro-magnetic induction, and especially for determining "electrodynamic capacities" in absolute measure.

‡ The mirror galvanometers commonly used in Germany have all much longer periods (ten or twenty times as long in many cases) for the vibration of their needles, and want proportionately longer contacts to obtain full advantage of their sensibility,—in each case a contact during a time equal to that of the vibration of the needle one way being required for this purpose.